SOLUCIONES DE LAS ACTIVIDADES

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1. a) $kx + C$

b) $\frac{x^6}{6} + C$

c) $\ln|x| + C$

d) $\text{sen } x + C$

e) $-\cos x + C$

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2. a) $\frac{x^5}{5} + C$

b) $\frac{3\sqrt[3]{x^4}}{4} + C$

c) $\frac{2\sqrt{x^5}}{5} + C$

d) $-\frac{1}{4x^4} + C$

e) $-\frac{3}{2\sqrt[3]{x^2}} + C$

f) $\frac{3\sqrt[3]{x^{10}}}{10} + C$

g) $-\frac{1}{5x^5} + C$

h) $\frac{5\sqrt[5]{x^{12}}}{12} + C$

i) $-\frac{1}{2x^2} + C$

3. a) $\frac{3^x}{\ln 3} + C$

b) $\frac{20^x}{\ln 20} + C$

c) $\frac{2^x}{\ln 2} + C$

4. a) $\frac{1}{\ln 2}(x \ln x - x) + C$

b) $\frac{1}{\ln 2}(-x \ln x + x) + C$

c) $\frac{2}{\ln 2}(x \ln x - x) + C$

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5. a) $\frac{x^4}{4} + \frac{4x^3}{3} + 3x + C$

b) $\frac{x^6}{6} - \frac{3x^4}{4} + \frac{5x^3}{3} - x^2 + 2x + C$

c) $x^4 - 2x^3 + \frac{3x^2}{2} - 11x + C$

d) $\frac{x^5}{5} - \frac{2x^3}{3} - 2x^2 - 3x + C$

6. a) $\int\left(x + \frac{1}{x^2}\right)dx = \frac{x^2}{2} - \frac{1}{x} + C$

b) $\int\left(2x + 1 + \frac{1}{x}\right)dx = x^2 + x + \ln|x| + C$

7. $\int f(x)dx = \ln|x| + C$

$P(e^2, 5) \rightarrow \ln e^2 + C = 5 \rightarrow 2 + C = 5 \rightarrow C = 3$

La primitiva que buscamos es :

$\ln x + 3$

8. $\int \cos x dx = \text{sen } x + C$

$P(\pi/2, 3) \rightarrow \text{sen } \pi/2 + C = 3 \rightarrow 1 + C = 3 \rightarrow C = 2$

La primitiva que buscamos es :

$\text{sen } x + 2$

9. a) $x + \frac{x^2}{2} + \frac{3\sqrt[3]{x^2}}{4} + C$

b) $-3\cos x + \frac{5}{x} + 4\sqrt[2]{x^3} + C$

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c) $\frac{1}{3}\text{sen } x + 3\cos x + 2e^x + \frac{\ln|x|}{2} + C$

10. a) $\int\left(-\frac{x}{4} - \frac{\ln x}{2}\right)dx = -\frac{x^2}{8} - \frac{\ln|x|}{2} + \frac{x}{2} + C$

b) $\int\left(2x^3 + 3x^2 + 2x + 3\right)dx = \frac{x^4}{2} + x^3 + x^2 + 3x +$

$+ C$

c) $\int\left(x^{5/2} + 3x^{3/2} - 4\sqrt{x} + \frac{5}{\sqrt{x}}\right)dx = \frac{2x^{7/2}}{7} +$

$+ \frac{6x^{5/2}}{5} - \frac{8x^{3/2}}{3} + 10\sqrt{x} \ + C$

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11. a) $2\int\left(x^2+1\right)2x\,dx = 2\frac{\left(x^2+1\right)^2}{2} + C = (x^2+1)^2 + C$

b) $\frac{1}{15}\int 3\cos(3x-5)dx = \frac{1}{15}\text{sen}(3x-5) + C$

c) $\frac{8}{3}\int 3x^2\sqrt{x^3+1}dx = \frac{16}{9}\left(x^3+1\right)^{3/2} + C$

d) $\int\left(3^{3x/2} - 3^{-x/2}\right)dx = \frac{2}{3}\int\frac{3}{2}3^{3x/2}dx +$

$+ 2\int\frac{3^{-x/2}}{-2}dx = \frac{2}{3}\cdot\frac{3^{3x/2}}{\ln 3} + 2\cdot\frac{3^{-x/2}}{\ln 3} + C$

e) $\int e^{-\left(1+x^3\right)}x^2dx = -\frac{1}{3}\int -3x^2e^{-\left(1+x^3\right)}dx =$

$= -\frac{1}{3}e^{-\left(x^3+1\right)} + C$

f) $\frac{4}{3}\int\frac{3x^2}{\left(x^3+6\right)^3}dx = -\frac{2}{3}\frac{1}{\left(x^3+6\right)^2} + C$

g) $4\int\frac{\cot g(x/4)}{4}dx = 4\left(\ln\left|\text{sen}\frac{x}{4}\right|\right) + C$

h) $\int\frac{e^{-x}}{1-e^{-x}}dx = \ln\left|1-e^{-x}\right| + C$

i) $-2\int\frac{tg(x/2)}{-2}dx = -2\ln\left|\cos\frac{x}{2}\right| + C$

j) $\int\left(e^x + e^{-x} - 2\right)dx = e^x - e^{-x} - 2x + C$

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12. a) $x + 6 = t \rightarrow dx = dt$

$$\int(t-6)t^{1/2}dt = \frac{2t^{5/2}}{5} - 4t^{3/2} + C =$$

$$= \frac{2(x+6)^{5/2}}{5} - 4(x+6)^{3/2} + C$$

b) $e^x = t \rightarrow e^x dx = dt$

$$\int\frac{3}{1+4t}dt = \frac{3\ln|1+4t|}{4} + C = \frac{3\ln\left|1+4e^x\right|}{4} + C$$

c) $x - 5 = t \rightarrow dx = dt$

$$\int(t+5)^2\sqrt{t}dt = \frac{2t^{7/2}}{7} + 4t^{5/2} + \frac{50t^{3/2}}{3} + C =$$

$$= \frac{2(x-5)^{2/7}}{7} + 4(x-5)^{5/2} + \frac{50(x-5)^{3/2}}{3} + C$$

d) $\text{sen } x = t \rightarrow \cos x\,dx = dt$

$$\int\frac{dt}{t^2} = -\frac{1}{t} + C = -\frac{1}{\text{sen } x} + C$$

e) $\sqrt{x} = t \rightarrow \frac{1}{2\sqrt{x}}\,dx = dt$

$$2\int e^t dt = 2e^t + C = 2e^{\sqrt{x}} + C$$

f) $1 - 2x^4 = t \rightarrow -8x^3\,dx = dt$

$$-\frac{1}{8}\int\frac{dt}{\sqrt{t}} = -\frac{\sqrt{t}}{4} + C = -\frac{\sqrt{1-2x^4}}{4} + C$$

13. a) $dx = 2 + 2tg^2\,t$

$$\int\frac{2+2tg^2t}{4tg^2t\sqrt{4tg^2t+4}}dt = \frac{1}{4}\int\frac{1+tg^2t}{tg^2t\sqrt{tg^2t+1}}dt =$$

$$= \frac{1}{4}\int\frac{\sqrt{tg^2t+1}}{tg^2t}dt = \frac{1}{4}\int\frac{\sqrt{\frac{1}{\cos^2 t}}}{\frac{\text{sen}^2t}{\cos^2 t}}dt =$$

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$$= \frac{1}{4}\int \frac{\cos t}{\operatorname{sen}^2 t}dt = -\frac{1}{4\operatorname{sen} t} + C =$$

$$= -\frac{1}{4\operatorname{sen}\left(\operatorname{arctg}\frac{x}{2}\right)} + C$$

b) $dt = dx / x$

$$\int \frac{2dt}{t} = 2\ln t + C = 2\ln|\ln x| + C$$

14. a) $\operatorname{arc\,sen}\frac{x}{2} = t$

$$\frac{dx}{\sqrt{4-x^2}} = dt$$

$$\int (4 - 4\operatorname{sen}^2 t)dt = 4\int \cos^2 t\,dt =$$

$= 2\cos t \operatorname{sen} t + 2t + C = \sqrt{1-x^2}\ (x/2) + 2 \operatorname{arc\,sen} x/2 + C$

b) $\operatorname{arc\,tg}\frac{x}{2} = t$

$$\frac{dx}{\sqrt{4-x^2}} = dt$$

$$\int \frac{2+2\operatorname{tg}^2 t}{2\operatorname{tg} t\sqrt{36+36\operatorname{tg}^2 t}}dt = \frac{1}{6}\int \frac{1+\operatorname{tg}^2 t}{\operatorname{tg} t\sqrt{1+\operatorname{tg}^2 t}}dt =$$

$$= \frac{1}{6}\int \frac{\sqrt{1+\operatorname{tg}^2 t}}{\operatorname{tg} t}dt = = \frac{1}{6}\int \frac{\sqrt{1+\operatorname{tg}^2 t}}{\operatorname{tg} t}dt =$$

$$= \frac{1}{6}\int \frac{1}{\operatorname{sen} t}dt = \frac{1}{6}\ln(\operatorname{cosec} t - \operatorname{cotg} t) + C =$$

$$= \frac{1}{6}\ln(\operatorname{cosec}(\operatorname{arc\,tg} x/2) - \operatorname{cotg}(\operatorname{arc\,tg} x/2)) + C$$

Nota: $\int \frac{1}{\operatorname{sen} t}dt$ se resuelve como la del *Ejemplo 7* del libro de texto, en este caso, se multiplica y divide por *cosec t − cotg t*.

c) $dx = \cos t\,dt$

$$\int \frac{(1+\operatorname{sen} t)^2 \cos t\,dt}{\sqrt{(1+\operatorname{sen} t)(1-\operatorname{sen} t)}}dt = \int (1+\operatorname{sen} t)^2 dt =$$

$$= \int (1 + \operatorname{sen}^2 t + 2\operatorname{sen} t)dt =$$

$$= \int \left(1 + \frac{1}{2} - \frac{\cos 2t}{2} + 2\operatorname{sen} t\right)dt =$$

$$= \frac{3t}{2} - \frac{\operatorname{sen} t}{4} - 2\cos t + C =$$

$$= \frac{3\operatorname{arcsen}(x-1)}{2} - \frac{x-1}{4} - 2\cos(\operatorname{arcsen}(x-1)) + C$$

d) $dx = dt / 2$

$$\frac{1}{2}\int \frac{8dt}{\sqrt{1-t^2}}dt = 4\int \frac{dt}{\sqrt{1-t^2}}dt = 4\operatorname{arcsen} t + C =$$

$= 4\operatorname{arc\,sen} 2x + C$

e) $dx = \frac{5\cos t}{4}$

$$5\int \frac{\sqrt{1-\operatorname{sen}^2 t}}{\operatorname{sen} t}\cos t\,dt = 5\int \frac{\cos^2 t}{\operatorname{sen} t}dt =$$

$$= 5\int \frac{1+\operatorname{sen}^2 t}{\operatorname{sen} t}dt = 5\int \frac{1}{\operatorname{sen} t}dt + 5\int \operatorname{sen} t\,dt =$$

$= 5\ln(\operatorname{cosec} t - \operatorname{cotg} t) - 5\cos t + C =$

$= 5\ln(\operatorname{cosec}(\operatorname{arc\,sen}(4x/5)) - \operatorname{cotg}(\operatorname{arc\,sen}(4x/5))) - 5\cos(\operatorname{arc\,sen}(4x/5)) + C$

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15. a) $\begin{cases} f(x) = \arctan x \\ dgx = x\,dx \end{cases} \rightarrow \begin{cases} df(x) = \frac{1}{x^2+1}dx \\ g(x) = \frac{x^2}{2} \end{cases}$

$$\int x \operatorname{arctg} x\,dx = \frac{x^2}{2}\arctan x - \int \frac{x^2}{2x^2+2}dx =$$

$$= \frac{x^2}{2}\arctan x - \int \left(\frac{1}{2} - \frac{1}{x^2+1}\right)dx =$$

$$= \frac{x^2}{2}\arctan x - \frac{x}{2} + \frac{\arctan x}{2} + C$$

b) $\begin{cases} f(x) = \arccos 2x \\ dgx = dx \end{cases} \rightarrow \begin{cases} df(x) = -\frac{2}{\sqrt{1-4x^2}}dx \\ g(x) = x \end{cases}$

$$\int \arccos 2x\,dx = x \cdot \arccos 2x + \int \frac{2x}{\sqrt{1-4x^2}}dx =$$

$$= x \cdot \text{arc}\cos 2x - \frac{\sqrt{1-4x^2}}{2} + C$$

c) $\begin{cases} f(x) = x^2 \\ dgx = \sqrt{1-x}\,dx \end{cases} \rightarrow \begin{cases} df(x) = 2xdx \\ g(x) = -\dfrac{2\sqrt{(1-x)^3}}{3} \end{cases}$

$$\int x^2\sqrt{1-x}\,dx = \frac{-2x^2\sqrt{(1-x)^3}}{3} + $$

$$+ \int \frac{4x\sqrt{(1-x)^3}}{3}dx$$

Calculamos ahora $\int \frac{4x\sqrt{(1-x)^3}}{3}dx$:

$\begin{cases} f(x) = 4x \\ dgx = \dfrac{\sqrt{(1-x)^3}}{3}dx \end{cases} \rightarrow \begin{cases} df(x) = 4dx \\ g(x) = -\dfrac{2\sqrt{(1-x)^5}}{15} \end{cases}$

Por lo tanto:

$$\int x^2\sqrt{1-x}\,dx = \frac{-2x^2\sqrt{(1-x)^3}}{3} -$$

$$- \frac{8x\sqrt{(1-x)^5}}{15} + \int \frac{8\sqrt{(1-x)^5}}{15}dx =$$

$$= \frac{-2x^2\sqrt{(1-x)^3}}{3} - \frac{8x\sqrt{(1-x)^5}}{15} -$$

$$- \frac{16\sqrt{(1-x)^7}}{105} + C$$

d) $\begin{cases} f(x) = 4x \\ dgx = \cos 2x \end{cases} \rightarrow \begin{cases} df(x) = 4dx \\ g(x) = \dfrac{\text{sen}2x}{2} \end{cases}$

$$\int 4x\cos 2xdx = \frac{4x\text{sen}2x}{2} - \int 2\text{sen}2xdx =$$

$$= 2x\text{sen}2x + \cos 2x + C$$

16. a) $\begin{cases} f(x) = e^x \\ dgx = \cos x \end{cases} \rightarrow \begin{cases} df(x) = e^x dx \\ g(x) = \text{sen}x \end{cases}$

$\begin{cases} f(x) = e^x \\ dgx = \text{sen}\ x \end{cases} \rightarrow \begin{cases} df(x) = e^x dx \\ g(x) = -\cos x \end{cases}$

$$\int e^x \cos xdx = e^x \cdot \text{sen}x + e^x \cdot \cos x - \int e^x \cdot \cos x$$

Por lo tanto, si $I = \int e^x \cos xdx$:

$$I = e^x \cdot \text{sen}x + e^x \cdot \cos x - I$$

Entonces:

$$I = \int e^x \cos xdx = \frac{e^x\text{sen}x + e^x\cos x}{2}$$

b) $\begin{cases} f(x) = \cos x \\ dgx = \cos x \end{cases} \rightarrow \begin{cases} df(x) = -\text{sen}\ xdx \\ g(x) = \text{sen}\ x \end{cases}$

$$\int \cos^2 xdx = \text{sen}\ x\cos x + \int \text{sen}^2 xdx =$$

$$= \text{sen}\ x\cos x + \int dx - \int \cos^2 xdx$$

$$2\int \cos^2 xdx = \text{sen}\ x\cos x + x + C;$$

$$\int \cos^2 xdx = \frac{\text{sen}\ x\cos x + x}{2} + C$$

c) Derivando sucesivamente x^3, se obtiene el resultado:

$$\int x^3\cos xdx = x^3\ \text{sen}\ x + 3x^2\cos x - 6\cos x -$$

$$- 6x\ \text{sen}\ x$$

d) $\begin{cases} f(x) = xe^x \\ dgx = \dfrac{1}{(x+1)^2} \end{cases} \rightarrow \begin{cases} df(x) = (1+x)e^x dx \\ g(x) = -\dfrac{1}{x+1} \end{cases}$

$$\int \frac{xe^x}{(x+1)^2}dx = -\frac{xe^x}{x+1} + \int e^x dx =$$

$$= -\frac{xe^x}{x+1} + e^x + C$$

e) $\begin{cases} f(x) = \ln x \\ dgx = \ln x \end{cases} \rightarrow \begin{cases} df(x) = \dfrac{dx}{x} \\ g(x) = x\ln x - x \end{cases}$

$$\int (\ln x)^2 dx = x(\ln x)^2 - x\ln x - \int (\ln x - 1)dx =$$

$$= x(\ln x)^2 - x\ln x - x\ln x + x + x + C =$$

$$= x(\ln x)^2 - 2x\ln x + 2x + C$$

f) $\begin{cases} f(x) = \ln x \\ dgx = x^3 \end{cases} \rightarrow \begin{cases} df(x) = \dfrac{dx}{x} \\ g(x) = \dfrac{x^4}{4}dx \end{cases}$

$$\int x^3\ln xdx = \frac{x^4\ln x}{4} - \int \frac{x^3}{4}dx = \frac{x^4\ln|x|}{4} - \frac{x^4}{16}$$

$$+ C$$

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17. a) $\frac{-1/2}{x}+\frac{3x/2}{x^2-2}$

b) $\frac{-1/4}{x}+\frac{3/8}{x-2}-\frac{1/8}{x+2}$

c) $\frac{3/4}{x-2}+\frac{5/4}{x+2}$

d) $\frac{-7/2}{x}+\frac{9x/2}{x^2-2}$

18. a) $\int\left(\frac{7x/4-1/2}{x^2}-\frac{3/4}{x-2}\right)dx=$

$$=\frac{1}{2x}+\frac{7\ln x}{4}-\frac{3\ln|x-2|}{4}+C$$

b) $\int\left(\frac{x^2}{2}+\frac{x}{4}+\frac{7}{8}+\frac{15+9x}{8(x-1)(2x+1)}\right)dx=$

$$=\int\left(\frac{x^2}{2}+\frac{x}{4}+\frac{7}{8}+\frac{1}{x-1}-\frac{7}{8(2x+1)}\right)dx=$$

$$=\frac{x^3}{6}+\frac{x^2}{8}+\frac{7x}{8}+\ln|x-1|-\frac{7\ln|2x+1|}{16}+C$$

c) $\int\left(x+\frac{3x^2-2x-1}{x^3-3x+2}\right)dx=$

$$=\int\left(x-\frac{5/3}{x+2}+\frac{4x/3-4/3}{(x-1)^2}\right)dx=$$

$$=\frac{x^2}{2}+\frac{5\ln|x+2|}{3}+\frac{4\ln|x-1|}{3}+C$$

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1. Difieren en una constante. Las gráficas son iguales salvo traslación vertical.

2. No se pueden cortar porque las funciones difieren en una constante, por lo tanto, la diferencia nunca podrá ser 0.

3. Porque la derivada de *ln x* es *1 / x* y, por lo tanto:

$$\int\frac{1}{x}dx=\ln x$$

4. Las gráficas de las primitivas de las funciones constantes son rectas mientras que las gráficas de las primitivas de las funciones polinómicas de primer grado son parábolas.

5. Una de las primitivas, la que pasa por (0, 1), de e^x e e^x.

Las gráficas de las primitivas de f(x) = 0 son paralelas al eje de abscisas.

6. $\int[f(x)+g(x)]dx=\int f(x)dx+\int g(x)dx$

$$\int kf(x)dx=k\int f(x)dx$$

Actividad personal.

7. Para calcular la integral $\int f(g(x))g'(x)dx$, con *f* y *g* funciones continuas, se hace el cambio g(x) = t y, por lo tanto, al tener que g'(x)dx = dt, la integral anterior se expresa de la siguiente forma:

$$\int f(t)dt$$

En el caso del ejemplo:

2tdt = 2xdx

$$\int t^2dt=\frac{t^3}{3}+C=\frac{\sqrt{(x^2-3)^3}}{3}+C$$

8. Se basa en la propiedad de las derivadas:

d[f(x)g(x)] = df(x)g(x) + f(x)dg(x)

Y, por lo tanto:

$$\int d[f(x)g(x)]dx=\int df(x)g(x)dx+\int f(x)dg(x)\,dx$$

Así que, finalmente:

$$\int f(x)dg(x)dx = f(x)g(x)-\int df(x)g(x)dx$$

Para calcular $\int x^2\cos 2x dx$ se deriva sucesivamente x^2 y se obtiene:

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$$\int x^2 \cos 2x dx = \frac{x^2}{2}\text{sen } 2x + \frac{x}{4}\cos 2x - \frac{1}{4}\text{sen } 2x$$

9. Se factoriza el polinomio del denominador. Las fracciones simples, cuya suma es la fracción original, tienen como denominador los factores de dicho polinomio:

$x^2 - 5x + 6 = (x - 2)(x + 3)$

$$\frac{A}{x-2} + \frac{B}{x-3} = \frac{2x-1}{x^2-5x+6};$$

$$\frac{A(x-3)+B(x-2)}{x^2-5x+6} = \frac{2x-1}{x^2-5x+6};$$

$$\begin{cases} A+B=2 \\ -3A-2B=-1 \end{cases} \rightarrow A=-3,\ B=5$$

$$\frac{2x-1}{x^2-5x+6} = \frac{-3}{x-2} + \frac{5}{x-3}$$

$$\int \frac{2x-1}{x^2-5x+6}dx = \int \frac{-3}{x-2}dx + \int \frac{5}{x-3}dx =$$

$= -3 \ln|x-2| + 5 \ln|x-3| + C$

10. a) $\frac{2x^3}{3} + \frac{3x^2}{2} + 5x + C$

b) $x^4 - x^2 + 3x + C$

c) $\frac{5x^3}{3} - \frac{3x^2}{2} + \ln|x| + C$

d) $4\sqrt{x^3} + \frac{1}{x} + C$

11. a) $\int \left(x^2 - \frac{x}{x^2}\right)dx = \frac{x^3}{3} - \ln|x| + C$

b) $\int \left(1 - \frac{4x}{4x^2+1}\right)dx = x - \frac{1}{2}\ln\left(4x^2+1\right) + C$

c) $\int \frac{x^3+5x^3-3x+2}{\sqrt{x}}dx = \int \frac{6x^3-3x+2}{\sqrt{x}}dx =$

$= \int \left(6x^{5/2} - 3x^{1/2} + \frac{2}{\sqrt{x}}\right)dx =$

$= \frac{12}{7}x^{7/2} - 2x^{3/2} + 4\sqrt{x} + C$

d) $\int \left(x - 2 + \frac{3x+3}{(x+1)^2}\right)dx = \frac{x^2}{2} - 2x + 3\ln|x+1| + C$

12. a) $\frac{4x^{3/4}}{3} + C$

b) $\sqrt{6x+1} + C$

c) $-\frac{1}{x^2+x+1} + C$

d) $\frac{2x^{5/2}}{5} - \frac{4x^{3/2}}{3} + 2\sqrt{x} + C$

13. a) $e^{x+3} + C$

b) $\frac{e^{4x}}{4} + C$

c) $\frac{-2e^{-5x/2}}{5} + C$

d) $\frac{e^{3x+1}}{3} + C$

e) $\frac{e^{3x-2}}{3} + C$

f) $\frac{-5e^{-2x^2}}{4} + C$

14. a) $\frac{1}{4}\ln|4x+1| + C$

b) $2x + 5 \ln|x-2| + C$

c) $\frac{x^2}{2} + \ln|x| + C$

d) $x - \text{arc tg } x + C$

e) $3\ln(x^2+1) + C$

f) $-\frac{1}{4}\frac{1}{\left(x^2+2\right)^2} + C$

15. a) $\frac{1}{3}\text{sen}(3x+5) + C$

b) $\frac{1}{5}\text{sen}(5x) + C$

c) $\frac{1}{3}\text{sen}^3 x + C$

d) $\frac{-2}{3}\cos(6x+2) + C$

e) $\frac{1}{4}\text{sen}(2x^2-3) + C$

f) $-4 \cos x - 2 \text{ sen } x + C$

g) $-\frac{1}{2}\cos(2\ln|x|)+C$

h) $-\frac{5}{2}\cos\left(\frac{1}{5}x^2\right)+C$

16. a) $\frac{1}{2}\frac{\text{sen}2x}{\cos 2x}+C$

b) $-\frac{1}{5}\frac{\text{sen}5x}{\cos 5x}+C$

17. a) $\frac{1}{3}\text{arcsen}3x+C$

b) $\text{arcsen}\frac{1}{4}x+C$

c) $4\text{arcsen}\,e^x+C$

d) $\text{arcsen}\left(\frac{1}{3}x-\frac{1}{3}\right)+C$

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18. a) $\text{arc tg}\,e^x+C$

b) $\frac{\sqrt{3}}{3}\text{arctg}\frac{\sqrt{3}x}{3}+C$

c) $\frac{1}{3}\text{arctg}x^3+C$

d) $\text{arctg}\frac{x}{2}+C$

e) $x-2\,\text{arc tg}\,x+C$

f) $-\text{arc tg}(\cos x)+C$

19. a) $x-\frac{x^2}{2}+x\ln|x|+C$

b) $2x+x^2-x\ln x+2x+C$

c) $2\ln(x+3)+C$

d) $\text{sen}\,x-5\ln x-\frac{1}{x}+C$

20. a) $\frac{x^6}{2}-4x^4+2x^3+8x^2-8x+C$

b) $\ln|\ln|x||+C$

c) $-2\cos\left(\sqrt{x}\right)+C$

d) $2\,\text{sen}\left(\sqrt{x+1}\right)$

e) $\ln(2-\cos x)+C$

f) $\ln^2 x+C$

g) $\frac{1}{2\cos^2 x}+C$

h) $-\sqrt{2\cos x+1}+C$

21. a) $\text{sen}\,x+\frac{\cos^4 x}{4}+C$

b) $-\frac{1}{5}\text{senx}\cos^4 x+\frac{1}{15}\cos^2 x\text{senx}+\frac{2}{15}\text{senx}+C$

22. a) $t=x-2$

$$\int\sqrt{x-2}\,dx=\frac{2(x-2)^{3/2}}{3}+C$$

b) $4-x^2=t$

$$\int x\sqrt{4-x^2}\,dx=-\frac{\left(4-x^2\right)^{3/2}}{3}+C$$

c) $x-1=t$

$$\int\frac{1}{(x-1)^2}dx=-\frac{1}{x-1}+C$$

d) $x+1=t$

$$\int\frac{x}{\sqrt{x+1}}dx=\frac{2(x+1)^{3/2}}{3}-2\sqrt{x+1}+C$$

e) $x=\sqrt{5}\,\text{sen}\,t$

$$\int\sqrt{5-x^2}\,dx=\frac{x}{\sqrt{5}}\cos\left(\text{arcsen}\frac{x}{\sqrt{5}}\right)+$$

$$+\frac{5}{2}\text{arcsen}\frac{x}{\sqrt{5}}+C$$

f) $x^2=t$

$$\int\sqrt{x^2-x^4}\,dx=-\frac{\left(1-x^2\right)^{3/2}}{3}+C$$

g) $\ln x=t$

$$\int\frac{dx}{x\sqrt{1-(\ln x)_2}}=\text{arc sen}(\ln|x|)+C$$

h) $x^2=t$

$$\int\text{senx}^2\,2x\,dx=-\cos x^2+C$$

SOLUCIONES DE LAS ACTIVIDADES

23. En cada caso, se deriva la parte polinómica sucesivamente:

a) $x^2e^x - 2xe^x + 2e^x + C$

b) $x^3e^x - 3x^2e^x + 6e^x - 6e^x + C$

c) $\frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$

d) $2xe^x - 3e^x + C$

e) $-x^2e^{-x} - 2xe^{-x} - 7e^{-x} + C$

f) $-x^2e^{-x} - 2xe^{-x} - 3e^{-x} + C$

24. a) $f(x) = \ln x$

$dg(x) = x\,dx$

$$\int x \ln x dx = \frac{x^2 \ln|x|}{2} - \frac{x^2}{4} + C$$

b) $f(x) = \ln x$

$dg(x) = x^2\,dx$

$$\int x^2 \ln x dx = \frac{x^3 \ln|x|}{3} - \frac{x^3}{9} + C$$

c) $f(x) = \ln (x + 1)$

$dg(x) = dx$

$$\int \ln(x+1)dx = \ln|x+1|(x+1) - x - 1 + C$$

d) $f(x) = \ln |x^2 + 1|$

$dg(x) = x\,dx$

$$\int x \ln(x^2+1)dx = \frac{(x^2+1)}{2}\ln|x^2+1| - \frac{x^2}{2} - \frac{1}{2} + C$$

e) $f(x) = \ln x$

$dg(x) = x \ln x\,dx$

$$\int x(\ln x)^2 dx = \frac{x^2}{2}(\ln|x|)^2 - \frac{x^2}{2}\ln|x| + \frac{x^2}{4} + C$$

f) $f(x) = \ln x$

$dg(x) = \frac{1}{x^2}dx$

$$\int \frac{\ln x}{x^2} dx = -\frac{\ln|x|}{x} - \frac{1}{x} + C$$

25. a) $f(x) = x^2$

$dg(x) = \text{sen } 2x\,dx$

$$\int x^2 \text{sen } 2x dx = -\frac{x^2}{2}\cos 2x + \frac{x}{2}\text{sen } 2x + \frac{1}{4}\cos 2x + C$$

b) $f(x) = e^{2x}$

$dg(x) = \text{sen } x\,dx$

$$\int e^{2x} \text{sen } x dx = -\frac{e^{2x}}{5}\cos x + \frac{2}{5}e^{2x}\cos x + C$$

c) Directa:

$$-\frac{1}{2}\cos x^2 + C$$

d) $f(x) = \text{sen } x$

$dg(x) = \text{sen } x\,dx$

$$\int \text{sen}^2 x dx = -\text{sen } x \cos x + \int \cos^2 x dx$$

Por otra parte:

$$\int \cos^2 x dx = \int dx - \int \text{sen}^2 x dx$$

Por lo tanto:

$$\int \text{sen}^2 x dx = -\text{sen } x \cos x + \int dx - \int \text{sen}^2 x dx\ ;$$

$$\int \text{sen}^2 x dx = -\frac{\text{sen } x \cos x}{2} + \frac{x}{2} + C$$

26. a) $f(x) = x$

$dg(x) = \cos x\,dx$

$$\int x \cos x\, dx = \cos x + x\text{sen}x + C$$

b) $f(x) = \text{sen } x$

$dg(x) = \text{sen } 3x\,dx$

$$\int \text{sen } x \text{ sen} 3x\, dx = \frac{\text{sen} 2x}{4} - \frac{\text{sen} 4x}{8} + C$$

c) $f(x) = \text{arc sen } x$

$dg(x) = dx$

$$\int \text{arc sen } x = x \text{ arc sen } x + \sqrt{1 - x^2} + C$$

d) $f(x) = x$

$$dg(x) = \frac{1}{\text{sen}^2 x}dx = \frac{\text{sen}^2 x + \cos^2 x}{\text{sen}^2 x}dx$$

$$\int \frac{x}{\text{sen}^2 x} dx = -x\frac{\cos x}{\text{sen } x} + \ln|\text{sen } x| + C$$

27. a) $f(x) = x - 1$

$dg(x) = \sqrt{x}\,dx$

$$\int (x-1)\sqrt{x}\,dx = \frac{2x^{5/2}}{5} - \frac{2x^{3/2}}{3} + C$$

b) $f(x) = \ln x$

$$dg(x) = \frac{1}{\sqrt{x}}\,dx$$

$$\int \frac{\ln x}{\sqrt{x}}\,dx = 2\ln x\sqrt{x} - 4\sqrt{x} + C$$

c) $f(x) = x$

$$dg(x) = \frac{x}{\sqrt{x+1}}\,dx$$

Tenemos , por lo tanto, que integrar:

$$\int \frac{x}{\sqrt{x+1}}\,dx$$

De nuevo debemos hacerlo por partes:

$f(x) = x$

$$dg(x) = \frac{1}{\sqrt{x+1}}\,dx$$

$$\int \frac{x}{\sqrt{x+1}}\,dx = 2x\sqrt{1+x} - \frac{4}{3}(1+x)^{3/2} + C$$

$$\int \frac{x^2}{\sqrt{x+1}}\,dx = 2x^2\sqrt{1+x} - \frac{4}{3}x(1+x)^{3/2} -$$

$$-\frac{4}{15}(1+x)^{5/2} + \frac{4}{3}(1+x)^{3/2} + C$$

d) $f(x) = \ln x$

$$dg(x) = \frac{4-2x^2}{x}\,dx$$

$$\int \left(\frac{4-2x^2}{x}\right)\ln x\,dx = 2\ln^2 x - x^2\ln x + \frac{x^2}{2} + C$$

28. a) $\ln x - \ln|1+x| + C$

b) $\frac{1}{2}\ln x - \frac{1}{2}\ln|2+x| + C$

c) $\ln\left|\frac{1-x}{1+x}\right| + C = -2\,\text{arc tgh}\,x + C$

d) $-\frac{1}{3(3x-4)} + C$

29. a) $\frac{3}{4}\ln|x-1| - \frac{1}{2(x+1)} - \frac{3}{4}\ln|x+1| + C$

b) $\frac{1}{3}x^3 + x^2 + 7x - \frac{9}{4}\ln|1+x| + \frac{61}{4}\ln|x-3| + C$

30. a) $F(x) = \frac{x^3}{6} + x + C$

$F(6) = 100 \rightarrow C = 58$

b) $F(x) = \frac{e^{2x+1}}{2} + C$

$F(-1/2) = 100 \rightarrow C = -e/2$

c) $F(x) = -x^2\cos x + 2\cos x + 2x\,\text{sen}\,x + C$

$F(\pi) = \pi^2 \rightarrow C = 2$

d) $F(x) = \frac{(x^2-3)^{3/2}}{3} + C$

$F(2) = 28/3 \rightarrow C = 9$

e) $F(x) = \frac{\ln^2 x}{6} + C$

$F(2) = 28/3 \rightarrow C = -\frac{1}{6} + \frac{13}{12}e^2$

31. $F(x) = \sec^2 x + C$

$F(\pi/4) = 1 \rightarrow F(x) = \sec^2 x - 1$

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32. $dx = (e^t + e^{-t})\,dt$

$$\int \frac{e^t + e^{-t}}{\sqrt{4 + (e^t - e^{-t})^2}}\,dt = \int \frac{e^t + e^{-t}}{\sqrt{(e^t + e^{-t})^2}}\,dt =$$

$$\int \frac{e^t + e^{-t}}{e^t + e^{-t}}\,dt = \int dt = t + C = \ln\frac{x + \sqrt{x^2+4}}{2} + C =$$

$= \text{arc senh}\,(x/2) + C$

33. a) $3\,\text{arc tg}\,(x+1) + C$

b) $\frac{1}{2}\ln(x^2 - 4x + 5) - \text{arc tg}\,(x-2) + C$

34. $f(x) = (x^2 - 2x + 2)\,e^x + C$

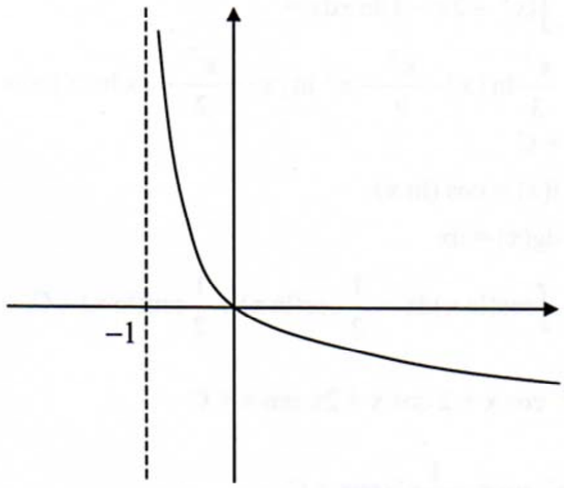
$f(0) = 2 \rightarrow C = 0$

35. $f(x) = a\ln(x+1) + C$

$f(0) = 1 \to a \ln 1 + + C = 1 \to C = 1$

$f(1) = -1 \to a \ln 2 + 1 = -1 \to a = -2 / \ln 2$

La función es $f(x) = \dfrac{-2}{\ln 2}\ln(x+1) + 1$



36. $f'(x) = \dfrac{x^2}{2}\ln x - \dfrac{x^2}{4} + C$

$f'(1) = 1 \to C = 5/4$

$f(x) = \dfrac{x^3}{6}\ln x - \dfrac{5}{36}x^3 + \dfrac{5}{4} + D$

$f(e) = e/4 \to D = -\dfrac{1}{36}e^3 - e$

37. $f'(x) = \ln(x^2+1) + C$

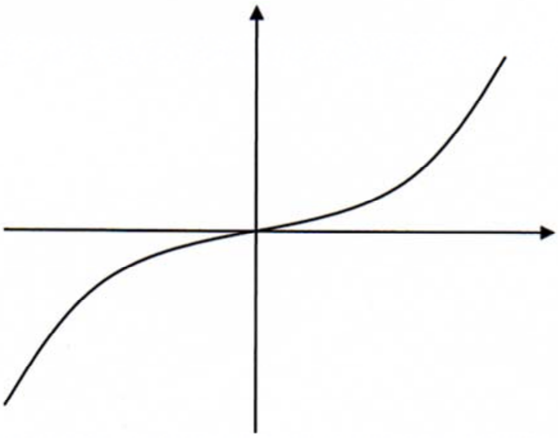
$f(x) = x\ln(x^2+1) - 2x + 2\text{arc tg } x + Cx + D$

$f(0) = 1 \to D = 1$

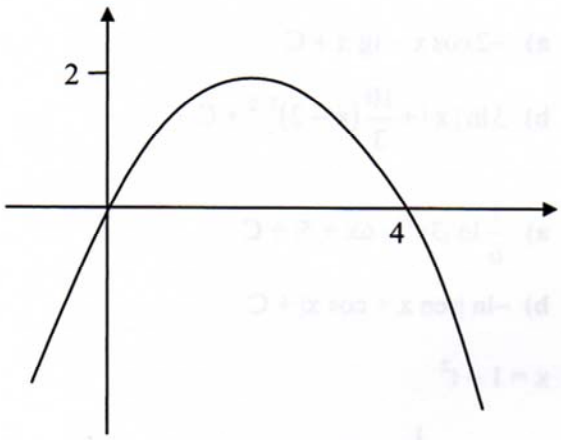
C puede tomar cualquier valor.

38. Representaremos las primitivas en el caso C = 0.

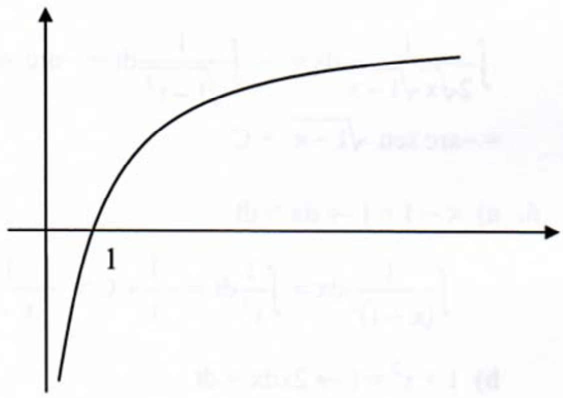
Superior izquierda $\to f'(x) = 3x^2 \to f(x) = x^3 + C$



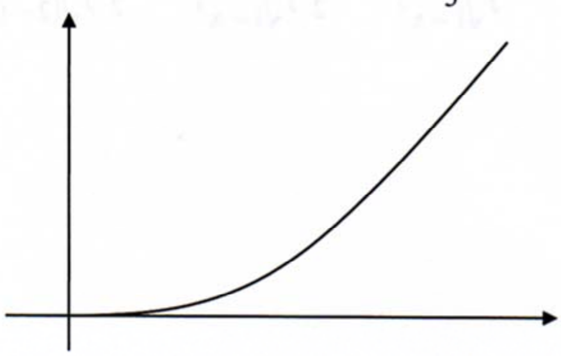
Superior derecha $\to f'(x) = -x + 2 \to f(x) = -x^2/2 + 2x + C$



Inferior izquierda $\to f'(x) = 1/x \to f(x) = \ln x + C$



Inferior derecha $\to f'(x) = \sqrt{x} \to f(x) = \dfrac{2}{3}\sqrt{x^3} + C$



Autoevaluación

1. $F(x) = \dfrac{x^3}{4} + x^2 - 3x + C$

$F(3) = 8 \to C = -1$

2. a) $\dfrac{x^4}{4} - \dfrac{5x^3}{3} + x^2 - 3x + C$

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SOLUCIONES DE LAS ACTIVIDADES

b) $\frac{x^2}{2} + 2x + \ln|x| + C$

3. a) $-2\cos x - \operatorname{tg} x + C$

b) $3\ln|x| + \frac{10}{3}(x-2)^{3/2} + C$

4. a) $\frac{1}{6}\ln|3x^2 - 6x + 5| + C$

b) $-\ln|\operatorname{sen} x + \cos x| + C$

5. $x = 1 - t^2$

$$dt = -\frac{1}{2\sqrt{1-x}}dx$$

$$\int \frac{1}{2\sqrt{x}\sqrt{1-x}}dx = -\int \frac{1}{\sqrt{1-t^2}}dt = -\operatorname{arc\ sen} t + C =$$
$$= -\operatorname{arc\ sen}\sqrt{1-x} + C$$

6. a) $x - 1 = t \rightarrow dx = dt$

$$\int \frac{1}{(x-1)^2}dx = \int \frac{1}{t^2}dt = -\frac{1}{t} + C = -\frac{1}{x-1} + C$$

b) $1 + x^2 = t \rightarrow 2xdx = dt$

$$\int \frac{x}{\sqrt{1-x^4}}dx = \frac{1}{2}\int \frac{2x}{\sqrt{1-x^4}}dx = \frac{1}{2}\int \frac{1}{\sqrt{(2-t)t}}dt =$$

$$= \operatorname{arc\ sen}(t-1) + C = \operatorname{arc\ sen} x^2 + C =$$

7. a) $f(x) = x^2 - 2x - 3$

$dg(x) = \ln x\, dx$

$$\int (x^2 - 2x - 3)\ln x dx =$$
$$\frac{x^3}{3}\ln|x| - \frac{x^3}{9} - x^2\ln|x| + \frac{x^2}{2} - 3x\ln|x| + 3x +$$
$$+ C$$

b) $f(x) = \cos(\ln x)$

$dg(x) = dx$

$$\int \cos(\ln x)dx = \frac{1}{2}\cos(\ln x) + \frac{1}{2}\operatorname{sen}(\ln x) + C$$

8. $-x^2\cos x + 2\cos x + 2x\operatorname{sen} x + C$

9. $\frac{1}{2}e^x\cos x + \frac{1}{2}e^x\operatorname{sen}x + C$

10. a) $\frac{x^3}{3} + \frac{5x^2}{2} + 2\ln|x-1| + C$

b) $\ln|x+1| - \frac{3}{x-1} + 2\ln|x-1| + C$

11. $f(x) = \frac{4x^3}{3}\ln|x| - \frac{4x^3}{9} + C$

$f(e) = -e^3/9 \rightarrow C = -e^3$